8. Probability

Definition and Concepts

The concept of probability is very vital in statistics but to define it satisfactorily is not an easy task. The probability may be defined in simple terms as follows–

"The probability of the happening of any one of the several equally likely events in the ratio of number of cases favourable to it to the total number of possible cases."

"Probability is a ratio or proportion which is expressed in terms of decimal or equation form." The probability of an impossible event is zero (0) because favourable conditions of its happening are nil, whereas the probability of fully certain event is 1 because all possible circumstances of its happening are favourable computation of probability depends upon its concept of interpretation. Thus, the experts of the subject differ very much as regards its different school of thoughts. On the basis of different points of view the following four school of thoughts are prevalent. On the basis of which different definitions of probability are given. The same is being discussed in the following pages

(1) Classical Approach of probability or Mathematical or A prior probability.

(2) Relative frequency approach or empirical or statistical probability.

(3) Axiomatic approach or Modern approach to probability.

1. Classical Approach : The classical approach of probability is the oldest & easiest. It was originated in 18th century. The basic assumption/approach for the classical theory is that outcomes of a random experiment are 'equally likely'.

> *According to Laplace,* "*Probability is the ratio of the number of 'favourable' cases to the total number of equally likely cases.*"

If probability of occurrence of N is denoted by p(N), then we have :

$$P = p(N) = \frac{\text{No. of favourable Cases}}{\text{Total number of equally likely cases}} = \frac{m}{m+n}$$

For example, if a coin is tossed, there are two equally likely results a head or a tail, hence the probability of head is $\frac{1}{2}$. Similarly if a six fixed dice is thrown the probability of coming 1, 2, 3, 4, 5, and 6 is 1/6th. As such the probability of receiving any number upward is 1/6th. Similarly in a pack of 52 playing cards, if one card is drawn at random, the probability of getting heart-Ace = 1/52 and getting any one Ace = 4/52 or 1/13 since the number of aces are four. The probability of getting any card of spade = 13/52 or 1/4. Happening of an event is called success or successful outcome and the notation for its expression are P or P(A). The probability of not happening of an event is called failure or unsuccessful outcome which is expressed by the notation q or $P(\bar{A})$.

Probability of happening of A	Probability of not happening of A
$P = P(A) = \frac{m}{m+n}$	$P = P(\bar{A}) = \frac{n}{m+n}$

Thus, it is clear that $\boxed{p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1}$

Similarly if P is known, q may be ascertained and if q is known p may be ascertained.

$$\boxed{\begin{array}{ll} p = 1 - q & q = 1 - p \\ P(A) = 1 - P(\bar{A}) & P(\bar{A}) = 1 - P(A) \end{array}}$$

For example, if a card is drawn of random from a pack of 52 cards, the probability of getting any King is 1/13 and that of not getting a king is 1 – $\frac{1}{13} = \frac{12}{13}$.

Process : The following are the steps for computation of probability :

(i) Total number of possibility of happening of an event is 'N' or m + n be calculated. All these results should be exhaustive, mutually exclusive or equally likely.

(ii) Find out the favourable number of that event, the probability for the happening of which is to be obtained.

(iii) Divide m by 'N' = m + n'. This is the required probability.

(iv) The measurement of probability is always between 0 and 1 limits :

$0 \leq p/(A) \leq 1$

If P(A) or P = 0 is the impossibility of happening of an event, then P(A) or P = 1 is the successful outcome.

For example : In principal contingent games there is no necessity of throwing a coin or a dice or betting or drawing a card from a pack. The probability is pre-determined without any experiment and argument, as such this concept is called a priori probability. For this we do not data for experiment or personal experience. Only on account of this reason the above concept is applicable mainly for contingent games.

2. Relative Frequency Approach or Empirical Approach : *According to empirical approach, the probability of an event represents the proportion of times under identical circumstances the result can be expected to occur.* The main assumptions are as under :

(i) The experiments are random, as there is no influence in any result or outcome, therefore all elements enjoy equal chance of selection.

(ii) There are large number of experiments.

> *According to **Von Mises,** "If the experiment be repeated a large number of times under essentially identical conditions, the limiting value of the ratio of the number of times of event. A happens to the total number of trails of the experiments as the number of trials increases identifinitely, is called the probability of the occurrence of A."*

Let an event A occurs m times in N repetitions of a random experiment. Then the ratio m/N, gives the relation frequency of the event. A and it will not vary appreciably from one trial to another. In the limiting case when N becomes sufficiently large it more or less settles to a number which is called the probability of A or

$$p(A) = \lim \frac{m}{N}$$

where N = ∞

The basis of statistical probability given desired results of stability of relative frequencies on the basis of increase in numerable experiments. For example, if a coin to tossed 100 times, the probability of its lying with head upward may be 60, the relative frequency of head will be 60/100 = 0.6. If it is again tossed 100 times, the coin lies with head upward 45 times, then in relative frequency will be $\frac{60+45}{100+100}\ \frac{105}{200}$ or 0.525. In this manner with the increase in no. of times of the being tossed the relative consistency will be attained and when the number is very large m/N will be equal to 0.5. This is the limit of relative frequency which is called empirical approach of consistency of probability. In the following figure, the number of trials increases to very large as such long term trend for consistency become clear.

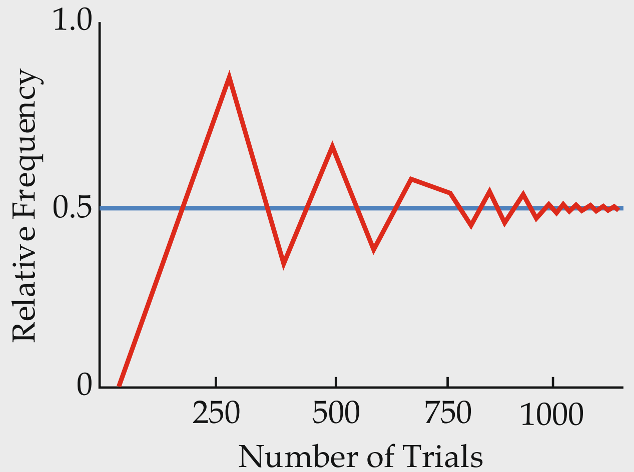

Fig. : *Consistency of Relative Frequency with The Increase in Number of Trials*

3. Discussion of Probability by means of Modern Approach : If 'n' is the probable result of a random trial and all the equally likely then in a sample survey 'S', there will be 'n' sample points and according to the above approach, the probability will be 1/n.

If 'm' are the sample points of an event A the probability of happening of 'A' will be :

Focus Formula

$$P(A) = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} \ldots (m\ times) = \frac{m}{n}$$

$$So \quad P(A) = \frac{No.\ of\ sample\ points\ in\ A}{Total\ No.\ of\ Sample\ points\ in\ A}\ or\ P(A) = \frac{n(A)}{n(s)} = \frac{n(A)}{N}$$

This formula is based upon the concept of classical approach to probability or empirical or statistical probability. It has become more useful, interesting and knowledgeable with the use of venn diagram, tree diagram and graphical presentation.

In short, it may be said that out of the above four approaches of probability, every one has its own usefulness. Infact, in a problem under study. We should use that approach which is easy and suitable for finding out the probability on the basis of which suitable decisions may be taken.

Bayes' Theorem – Inverse Probability

According to Bayes theorem, probabilities price to posterior probabilities are known as Priori or Prior Probabilities. The reason is that computation of such probabilities can be done without considering the information about the sample. *As per Bayes' theorem prior probabilities can be determined, again on the basis of the sample information. These are called Posteriori or Posterior probabilities. Posterior probabilities are always conditional probabilities shall become prio-probabilities and on the basis of these, revised probabilities can be calculated again.* Posterior probabilities are conditional probabilities, but these are different from general conditional probabilities. General conditional probability means the probability of an experiment given the value of a condition, whereas posterior probability is the probability of the value of that condition when the result of the experiment is already known. Hence, it is also regarded as inverse probability.

Example : The probabilities of appointment of one of the three persons–a politician, a retired I.A.S. officer and an educationist – as the vice-chancellor of a University are 35 percent, 25 percent and 40 percent respectively. The probabilities that standard research activities will be encouraged and promoted by them if they are appointed, are 0.4, 0.8 and 0.9 respectively. What is the probability that research will be promoted by the new vice-chancellor ?

Solution : Let probabilities of appointment as a Vice chancellor of a university of a politician, retired I.A.S. Officer and an educationist be expressed by notation A_1, A_2 and A_3 the event of research activities be taken as B. Thus, Given – $P(A_1) = 0.35$, $P(A_2) = 0.25$, $P(A_3) = 0.40$.

Conditional Probabilities and Joint Probabilities may be calculated in following table :

Table Showing Conditional and Joint Probabilities

(i)	(ii)	(iii)	(iv)
Event	Prior Probability	Conditional Probability of B given A_i	Joint Probability $P(A_i) \times P(B/A_i)$
A_i	$P(A_i)$	$P(B/A_i)$	$P(A_i \cap B)$
A_1	0.35	0.4	0.140
A_2	0.25	0.8	0.200
A_3	0.40	0.9	0.360
Total	1.00		P(B) = 0.700

Conditional Probabilities – $P(B/A_1) = 0{\cdot}4$; $P(B/A_2) = 0{\cdot}8$; $P(B/A_3) = 0{\cdot}9$

Research promotion by mutually exclusive ways as under –

(a) A politician is appointed & encourage research - $P(A_1 \cap B)$

$$P(A_1 \cap B) = P(A_1) \times P(B/A_1)$$

(b) A retired I.A.S. is appointed and encourage – $(A_2 \cap B)$:

$$P(A_2 \cap B) = P(A_2) \times P(B/A_2)$$

(c) An educationalist is appointed and encourage research – $(A_3 \cap B)$;

$$P(A_3 \cap B) = P(A_3) \times P(B/A_3)$$

Thus probabilities of encouraging research working by a new vice-chancellor–

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$
$$= P(A_1) \times P(B/A_1) + P(A_2) \times P(B/A_2) + P(A_3) \times P(B/A_3)$$
$$= 0.140 + 0.200 + 0.360$$
$$= (0.35 \times 0.4) + (0.25 \times 0.8) + (0.4 \times 0.9) = 0.7$$